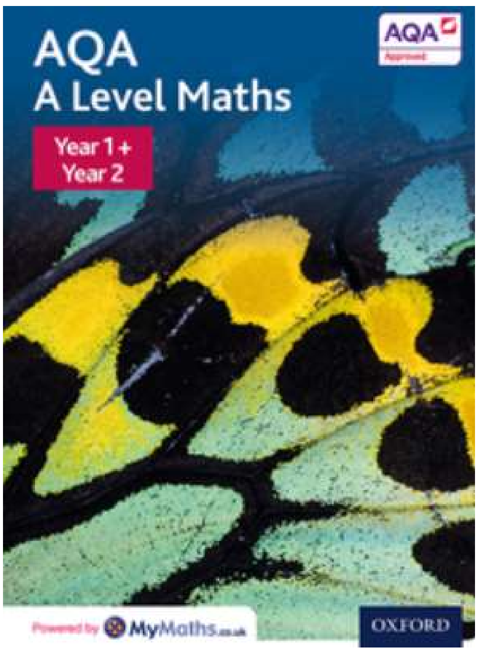
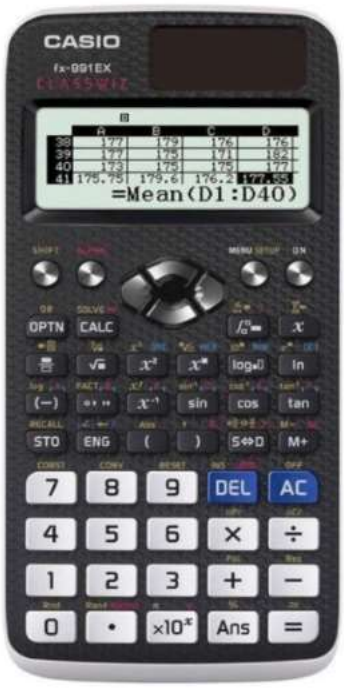
The Redhill Academy

A Level Maths
(2023 – 2025)

Summer Work





Expectations

The material in this booklet should be familiar to you from your GCSE work. It forms the building blocks for much of the A Level course; **fluency and confidence in these topics is crucial to survival at A Level.**

We expect you to work through this booklet **thoroughly** over the summer. Use the examples as a guide and complete all the exercises. We recommend you do a little work often rather than cramming it in at the end.

We expect you to bring your booklet with you to the first lesson of Year 12.

There will be an assessment on this material early in the term. Failing this assessment will result in further work being set and we may ultimately ask you to reconsider your options if we do not feel you will be successful.

Remember you can use any GCSE revision resources you may have including MathsWatch and revision guides.

Please also consult the subject reading list provided for further resources and suggestions to keep you occupied over the summer!

We look forward to seeing you in September.

Miss N Connolly

KS5 Curriculum Area Leader

Topic A: Indices and surds

You can apply the rules of indices and surds to simplify algebraic expressions. The following expressions can be simplified in **index form**:

Key point

$x^a \times x^b = x^{a+b}$ $\quad x^a \div x^b = x^{a-b}$ $\quad (x^a)^b = x^{ab}$

Example 1

Simplify these expressions. **a** $2x^3 \times 3x^5$ **b** $12x^7 \div 4x^3$ **c** $(3x^5)^3$

a $2x^3 \times 3x^5 = 6x^{3+5}$

$= 6x^8$

Multiply the coefficients together and use $x^a \times x^b = x^{a+b}$

b $12x^7 \div 4x^6 = \dfrac{12x^7}{4x^6}$

$= 3x$

Since $\frac{12}{4} = 3$ and $x^a \div x^b = x^{a-b}$ so $\frac{x^7}{x^6} = x^1$ which we just write as x

c $(3x^5)^3 = 3^3(x^5)^3$

Both the 3 and the x^5 must be raised to the power 3

$= 27x^{15}$

Since $(x^a)^b = x^{ab}$

Roots can also be expressed using indices, such that the square root of x is written as $\sqrt{x} = x^{\frac{1}{2}}$

In general:

Key point

The nth root of x is written $\sqrt[n]{x} = x^{\frac{1}{n}}$, and this can be raised to a power to give $\sqrt[n]{x^m} = x^{\frac{m}{n}}$

Key point

A power of -1 indicates a reciprocal, so $x^{-1} = \dfrac{1}{x}$ and, in general, $x^{-n} = \dfrac{1}{x^n}$

Example 2

Evaluate each of these without using a calculator.

a $25^{0.5}$ **b** 6^{-2} **c** $8^{\frac{2}{3}}$

a $25^{0.5} = 25^{\frac{1}{2}}$

$= \sqrt{25}$

Since a power of $\frac{1}{2}$ represents a square root.

$= 5$

b $6^{-2} = (6^2)^{-1}$

$= \dfrac{1}{6^2}$

Since a power of -1 represents a reciprocal.

$= \dfrac{1}{36}$

c $8^{\frac{2}{3}} = \left(8^{\frac{1}{3}}\right)^2$

Always calculate a root before a power.

$= 2^2$

Since the cube root of 8 is 2

$= 4$

Task:

Evaluate each of these without using a calculator.

a $49^{\frac{1}{2}}$ **b** $27^{\frac{1}{3}}$ **c** 5^{-1} **d** $64^{-\frac{1}{3}}$

e $9^{\frac{3}{2}}$ **f** $16^{\frac{3}{4}}$ **g** $125^{-\frac{2}{3}}$ **h** $\left(\frac{1}{2}\right)^3$

i $\left(\frac{1}{9}\right)^{-2}$ **j** $\left(\frac{4}{9}\right)^{\frac{1}{2}}$ **k** $\left(\frac{9}{16}\right)^{-0.5}$ **l** $\left(\frac{27}{8}\right)^{-\frac{2}{3}}$

Example 3

Write these expressions in simplified index form.

a $\sqrt[3]{x}$ **b** $\frac{2}{x^3}$ **c** $\frac{2x}{\sqrt{x}}$

a $\sqrt[3]{x} = x^{\frac{1}{3}}$

b $\frac{2}{x^3} = 2x^{-3}$

c $\frac{2x}{\sqrt{x}} = \frac{2x}{x^{\frac{1}{2}}}$

$= 2x^{1-\frac{1}{2}}$

$= 2x^{\frac{1}{2}}$

Since $\sqrt{x} = x^{\frac{1}{2}}$

Subtract the powers, remembering that $x = x^1$

Task:

Write each of these expressions in simplified index form.

a $x^3 \times x^7$ **b** $7x^5 \times 3x^6$ **c** $5x^4 \times 8x^7$ **d** $x^8 \div x^2$

e $8x^7 \div 2x^9$ **f** $3x^8 \div 12x^7$ **g** $(x^5)^7$ **h** $(x^2)^{-5}$

i $(3x^2)^4$ **j** $(6x^5)^2$ **k** $\sqrt{x^3}$ **l** $\sqrt[4]{x^5}$

m $\frac{5\sqrt{x}}{x}$ **n** $2x\sqrt{x}$ **o** $\frac{x^2}{3\sqrt{x}}$ **p** $x^3(x^5 - 1)$

q $x^3(\sqrt{x} + 2)$ **r** $\frac{x+2}{x^3}$ **s** $\frac{\sqrt{x}+3}{x}$ **t** $\frac{(3-x^3)}{\sqrt{x}}$

A **surd** is an **irrational** number involving a root, for example $\sqrt{2}$ or $\sqrt[3]{7}$.
You can multiply and divide surds using the rules:

An irrational number is a real number that cannot be written as a fraction $\frac{a}{b}$, where a and b are integers with $b \neq 0$

Key point

$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ and $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

You can simplify surds by finding square-number factors, for example $\sqrt{12} = \sqrt{4}\sqrt{3} = 2\sqrt{3}$. It may also be possible to simplify expressions involving surds by collecting like terms or by **rationalising the denominator**. Rationalising the denominator means rearranging the expression to remove any roots from the denominator.

Key point

To rationalise the denominator, multiply both the numerator and denominator by a suitable expression:

$\frac{1}{\sqrt{a}} \times \frac{\sqrt{a}}{\sqrt{a}} = \frac{\sqrt{a}}{a}$ (multiply numerator and denominator by $\sqrt{a}$)

$\frac{1}{a+\sqrt{b}} \times \frac{a-\sqrt{b}}{a-\sqrt{b}} = \frac{a-\sqrt{b}}{a^2-b}$ (multiply numerator and denominator by $a-\sqrt{b}$)

$\frac{1}{a-\sqrt{b}} \times \frac{a+\sqrt{b}}{a+\sqrt{b}} = \frac{a+\sqrt{b}}{a^2-b}$ (multiply numerator and denominator by $a+\sqrt{b}$)

Example 4

Simplify these expressions without using a calculator.

a $\sqrt{18}+5\sqrt{2}$ **b** $\frac{6}{\sqrt{3}}$ **c** $\frac{2}{1-\sqrt{5}}$

a $\sqrt{18} = \sqrt{9}\sqrt{2}$
$= 3\sqrt{2}$

9 is a square-number factor of 18 so you can simplify $\sqrt{18}$

Therefore $\sqrt{18}+5\sqrt{2} = 3\sqrt{2}+5\sqrt{2}$
$= 8\sqrt{2}$

Collect like terms.

b $\frac{6}{\sqrt{3}} = \frac{6\sqrt{3}}{\sqrt{3}\sqrt{3}}$

Rationalise the denominator by multiplying numerator and denominator by $\sqrt{3}$

$= \frac{6\sqrt{3}}{3}$
$= 2\sqrt{3}$

Since $6 \div 3 = 2$

c $\frac{2}{1-\sqrt{5}} = \frac{2(1+\sqrt{5})}{(1-\sqrt{5})(1+\sqrt{5})}$

Rationalise the denominator by multiplying numerator and denominator by $1+\sqrt{5}$

$= \frac{2(1+\sqrt{5})}{-4}$

$(1-\sqrt{5})(1+\sqrt{5}) = 1-\sqrt{5}+\sqrt{5}-5$
$= 1-5 = -4$

$= -\frac{1}{2}(1+\sqrt{5})$

Task:

Simplify these expressions fully without using a calculator.

a $\sqrt{8}$ **b** $\sqrt{75}$ **c** $2\sqrt{24}$ **d** $3\sqrt{48}$

e $\sqrt{20}+\sqrt{5}$ **f** $\sqrt{27}-\sqrt{12}$ **g** $5\sqrt{32}-3\sqrt{8}$ **h** $\sqrt{50}+3\sqrt{125}$

Simplify these expressions fully without using a calculator.

a $\frac{1}{\sqrt{7}}$ **b** $\frac{2}{\sqrt{8}}$ **c** $\frac{12}{\sqrt{3}}$ **d** $\frac{\sqrt{8}}{\sqrt{12}}$

e $\frac{1}{1+\sqrt{3}}$ **f** $\frac{2}{1+\sqrt{2}}$ **g** $\frac{8}{1-\sqrt{5}}$ **h** $\frac{2}{\sqrt{5}-1}$

Expand the brackets and fully simplify each expression.

a $(1+\sqrt{2})(3+\sqrt{2})$ **b** $(1+\sqrt{2})(3-\sqrt{2})$ **c** $(1-\sqrt{2})(3+\sqrt{2})$ **d** $(1-\sqrt{2})(3-\sqrt{2})$

Topic B: Expanding brackets

You know how to find the product of two binomials by multiplying every combination of terms together and simplifying. Take extra care when squaring a binomial, and remember that $(x+a)^2 = (x+a)(x+a) = x^2 + 2ax + a^2$ NOT $x^2 + a^2$

$(ax+b)(cx+d)$

Example 1

Expand and simplify $(3x-5)^2$

$(3x-5)^2 = (3x-5)(3x-5)$ — Always write in this form until you are confident.

$= 9x^2 - 15x - 15x + 25$

$= 9x^2 - 30x + 25$ — Simplify the x-terms.

To find the product of three binomials, first expand any pair, then multiply by the third.

Example 2

Expand and simplify $(x+3)(x-2)(x+1)$

$(x+3)(x-2) = x^2 - 2x + 3x - 6$ — Expand the first two pairs.

$= x^2 + x - 6$ — Simplify $-2x + 3x$ to x

$(x^2 + x - 6)(x+1) = x^3 + x^2 + x^2 + x - 6x - 6$ — 3 terms × 2 terms = 6 terms

$= x^3 + 2x^2 - 5x - 6$ — Add the x^2-terms and simplify $x - 6x$ to $-5x$

Task:

Expand and simplify each of these expressions.

a $(x-4)^2$ **b** $(x+6)^2$ **c** $(x-9)^2$

g $(4x+3)^2$ **h** $(5x+2)^2$ **i** $(3-x)^2$

Expand and simplify each of these expressions.

a $(x+5)(x+2)(x+4)$

d $(x+6)(2x-5)(x-8)$

g $(x+5)^2(x+9)$

j $(2x+3)^2(4-x)$

Topic C: Solving linear equations and rearranging formulae

This topic recaps the **balance** method to solve problems involving linear equations, and both the **elimination** and **substitution** methods to solve linear simultaneous equations.

You can solve linear equations and inequalities using the **balance** method where the same operation is applied to both sides.

Example 1

Solve the equation $7x-5=3x-2$

$4x-5=-2$ — Subtract $3x$ from both sides of the equation.

$4x=3$ — Add 5 to both sides of the equation.

$x=\frac{3}{4}$ — Divide both sides of the equation by 4

Example 2

Solve the inequality $5(x-2)\leq 2x+1$

$5x-10\leq 2x+1$ — First expand the brackets.

$3x-10\leq 1$ — Subtract $2x$ from both sides.

$3x\leq 11$ — Add 10 to both sides.

$x\leq\frac{11}{3}$ — Divide both sides by 3

Task:

Solve each of these linear equations.

a $3(2x+9)=7$ **b** $7-3x=12$ **c** $\frac{x+4}{5}=7$ **d** $2x+7=5x-6$

e $8x-3=2(3x+1)$ **f** $\frac{2x+9}{12}=x-1$

Solve each of these linear inequalities.

a $\frac{x}{2}+7\geq 5$ **b** $3-4x<15$ **c** $5(x-1)>12+x$ **d** $\frac{x+1}{3}>2$

e $8x-1\leq 2x-5$ **f** $3(x+1)\geq\frac{x-3}{2}$

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When solving inequalities, remember that multiplying or dividing by a negative number will reverse the inequality sign. For example, $5 > 3$ but $-5 < -3$

Equations and formulae can be rearranged using the same method as for solving equations.

Example 3

Rearrange $Ax - 3 = \dfrac{x+B}{2}$ to make x the subject.

$2Ax - 6 = x + B$ — Multiply both sides by 2

$2Ax - 6 - x = B$ — Subtract x from both sides.

$2Ax - x = B + 6$ — Add 6 to both sides.

$x(2A - 1) = B + 6$ — Factorise the side involving x

$x = \dfrac{B+6}{2A-1}$ — Divide both sides by $(2A - 1)$ to make x the subject.

Task:

Rearrange each of these formulae to make x the subject.

a $2x + 5 = 3A - 1$ **b** $x + u = vx + 3$ **c** $\dfrac{3x-1}{k} = 2x$ **d** $5(x - 3m) = 2nx - 4$

You can solve linear simultaneous equations using the **elimination** method, as shown in Example 4. The solutions to simultaneous equations give the point of intersection between the lines represented by the two equations.

Example 4

Solve the simultaneous equations $5x-4y=17$, $3x+8y=5$

$15x+40y=25$ (1) — Multiply the second equation by 5

$15x-12y=51$ (2) — Multiply the first equation by 3

$(1)-(2): 52y=-26$ — Subtract equation (2) from equation (1) to eliminate x

$y=-\frac{1}{2}$

$5x-4\left(-\frac{1}{2}\right)=17$ — Substitute $y=-\frac{1}{2}$ into one of the original equations.

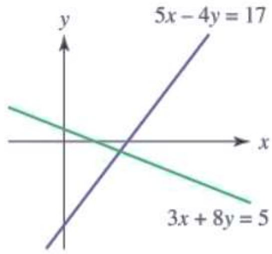
$5x+2=17$ — Solve this equation to find the value of x

$5x=15$

$x=3$

The example shows you that the lines $5x-4y=17$ and $3x+8y=5$ intersect at the point $\left(3,-\frac{1}{2}\right)$

If you are given the equation of two lines where y is the subject then the easiest way to solve these simultaneously is to use the **substitution** method as shown in the next example.



Task:

Use algebra to solve each of these pairs of simultaneous equations.

a $5x+12y=-6,\ x+5y=4$ **b** $7x+5y=14,\ 3x+4y=19$ **c** $2x-5y=4,\ 3x-8y=5$

Example 5

Find the point of intersection between the lines with equations $y=2x+5$ and $y=7-3x$

$2x+5=7-3x$

$5x+5=7$

$5x=2$

$x=0.4$

$y=2(0.4)+5$

$=5.8$

So the lines intersect at the point (0.4, 5.8)

Substitute $2x+5$ for y in the second equation.

Solve to find the value of x

Substitute $x=0.4$ into either of the original equations to find the y-coordinate.

Task:

Use algebra to find the point of intersection between each pair of lines.

a $y=8-3x$, $y=2-5x$ **b** $y=7x-4$, $y=3x-2$ **c** $y=2x+3$, $y=5-x$

Topic D: Factorising quadratics

Expressions such as $5x^2+x$, $2x^2+4$ and x^2+2x-1 are called **quadratics** and can sometimes be factorised into two linear factors. There are three types of quadratics to consider:

1. Quadratics of the form ax^2+bx have a common factor of x so can be factorised using a single bracket and removing the highest common factor of the two terms, e.g. $6x^2+8x=2x(3x+4)$
2. Quadratics of the form x^2+bx+c will sometimes factorise into two sets of brackets. You need to find two constants with a product of c and a sum of b, e.g. $x^2-3x+2=(x-2)(x-1)$ since $-2\times-1=2$ and $-2+-1=-3$
3. Quadratics of the form ax^2-c will factorise if a and c are square numbers. This is called the **difference of two squares**, e.g. $4x^2-9=(2x+9)(2x-9)$

Example 1

Factorise each of these quadratics.

a $9x^2+15x$ **b** $x^2+3x-10$ **c** x^2-16

a $9x^2+15x=3x(3x+5)$

The highest common factor of $9x^2$ and $15x$ is $3x$

b $x^2+3x-10=(x+5)(x-2)$

You need to find two constants with a product of -10 and a sum of 3: $5\times-2=-10$ and $5+-2=3$ so the constants are -2 and 5

c $x^2-16=(x+4)(x-4)$

x^2 and 16 are both square numbers.

Task:

Fully factorise each of these quadratics.

a $3x^2+5x$ **b** $8x^2-4x$ **c** $17x^2+34x$ **d** $18x^2-24x$

Factorise each of these quadratics.

a x^2-100 **b** x^2-81 **c** $4x^2-9$ **d** $64-9x^2$

Factorise each of these quadratics.

a x^2+5x+6 **b** $x^2-7x+10$ **c** x^2-5x-6 **d** $x^2+3x-28$

When factorising quadratics of the form ax^2+bx+c with $a\neq 1$, first split the bx term into two terms where the coefficients multiply to give the same value as $a\times c$

Example 2

Factorise each of these quadratics.

a $3x^2+11x+6$ **b** $2x^2-9x+10$

a $3x^2+11x+6=3x^2+9x+2x+6$ — Split $11x$ into $9x+2x$ since $9\times 2=18$ and $3\times 6=18$

$=3x(x+3)+2(x+3)$ — Factorise the first pair of terms and the second pair of terms.

$=(3x+2)(x+3)$

b $2x^2-9x+10=2x^2-4x-5x+10$ — Split $9x$ into $-4x-5x$ since $-4\times -5=20$ and $2\times 10=20$

$=2x(x-2)-5(x-2)$ — Factorise the first pair of terms and the second pair of terms.

$=(2x-5)(x-2)$

Task:

Factorise each of these quadratics.

a $3x^2+7x+2$ **b** $6x^2+17x+12$ **c** $4x^2-13x+3$ **d** $2x^2-7x-15$

Example 3

Use factorisation to find the roots of these quadratic equations.

a $4x^2+12x=0$ **b** $5x^2=21x-4$

a $4x^2+12x=4x(x+3)$ — Factorise the quadratic.

$4x(x+3)=0\Rightarrow 4x=0$ or $x+3=0$ — One of the factors must be equal to zero.

If $4x=0$ then $x=0$ and if $x+3=0$ then $x=-3$ — Solve to find the roots.

b $5x^2-21x+4=0$ — Rearrange so you have a quadratic expression equal to zero.

$5x^2-21x+4=5x^2-20x-x+4$ — Write $-21x=-x-20x$ since $-20\times -1=20$ and $5\times 4=20$

$=5x(x-4)-(x-4)$

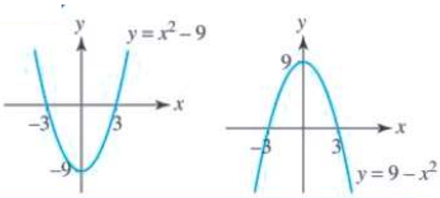
$=(5x-1)(x-4)$ — Factorise the quadratic.

$(5x-1)(x-4)=0\Rightarrow 5x-1=0$ or $x-4=0$ — The product is zero so one of the factors must be equal to zero.

If $5x-1=0$ then $x=\frac{1}{5}$ and if $x-4=0$ then $x=4$ — Solve to find the roots.

A quadratic function has a **parabola** shaped curve.

When you sketch the graph of a quadratic function you must include the coordinates of the points where the curve crosses the x and y axes.



Example 4

Sketch these quadratic functions.

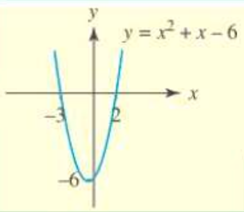
a $y = x^2 + x - 6$ **b** $y = -x^2 + 4x$

a When $x = 0$, $y = -6$

When $y = 0$, $x^2 + x - 6 = 0$

$x^2 + x - 6 = (x + 3)(x - 2)$

$(x + 3)(x - 2) = 0 \Rightarrow x = -3$ or $x = 2$

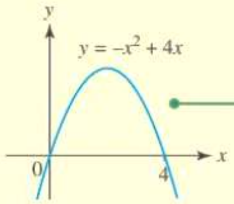


b When $x = 0$, $y = 0$

When $y = 0$, $-x^2 + 4x = 0$

$-x^2 + 4x = -x(x - 4)$

$-x(x - 4) = 0 \Rightarrow x = 0$ or $x = 4$



Find the y-intercept by letting $x = 0$

Find the x-intercept by letting $y = 0$

Factorise to find the roots.

Sketch the parabola and label the y-intercept of -6 and the x-intercepts of -3 and 2

Sketch the parabola, it will be this way up since the x^2 term in the quadratic is negative. Label the x and y intercepts.

Factorise to find the roots.

Find the y-intercept by letting $x = 0$

Find the x-intercept by letting $y = 0$

Task:

Sketch each of these quadratic functions, labelling where they cross the x and y axes.

a $y = x^2 + 6x$ **b** $y = 3x^2 - 12x$ **c** $y = x^2 - 121$ **d** $y = x^2 - 3x - 10$

Topic E: Completing the square

Some quadratics are **perfect squares** such as $x^2-8x+16$ which can be written $(x-4)^2$. For other quadratics you can **complete the square**. This means write the quadratic in the form $(x+q)^2+r$

Key point

The completed square form of x^2+bx+c is $\left(x+\frac{b}{2}\right)^2-\left(\frac{b}{2}\right)^2+c$

If you have an expression of the form ax^2+bx+c then first factor out the a, as shown in Example 1

Example 1

Write each of these quadratics in the form $p(x+q)^2+r$ where p, q and r are constants to be found.

a x^2+6x+7 **b** $-2x^2+12x$

a $x^2+6x+7=\left(x+\frac{6}{2}\right)^2-\left(\frac{6}{2}\right)^2+7$

$=(x+3)^2-9+7=(x+3)^2-2$

b $-2x^2+12x=-2\left[x^2-6x\right]$

$=-2\left[(x-3)^2-9\right]=-2(x-3)^2+18$

The constant term in the bracket will be half of the coefficient of x

First factor out the coefficient of x^2 then complete the square for the expression in the square brackets.

Task:

Write each of these quadratic expressions in the form $p(x+q)^2+r$

a x^2+8x **b** x^2-18x

e $x^2-7x+10$ **f** x^2+5x+9

i $2x^2-10x+3$ **j** $-x^2+12x-1$

Topic F: Quadratic formula

You can solve a quadratic equation using the **quadratic formula**. The quadratic formula can also be used to quickly determine how many roots a quadratic equation has.

Key point

The quadratic formula for $ax^2+bx+c=0$ is $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

Example 1

Solve the equation $3x^2-5x-7=0$ using the quadratic formula.

$a=3, b=-5, c=-7$

$x=\dfrac{-(-5)\pm\sqrt{(-5)^2-4\times3\times(-7)}}{2\times3}$

$=\dfrac{5\pm\sqrt{109}}{6}$

$=2.57$ or -0.91 (to 2 dp)

Substitute into the formula, taking care with negatives.

Use your calculator to give answer as a decimal:
$\dfrac{5+\sqrt{109}}{6}=2.57$ and
$\dfrac{5-\sqrt{109}}{6}=-0.91$

Task:

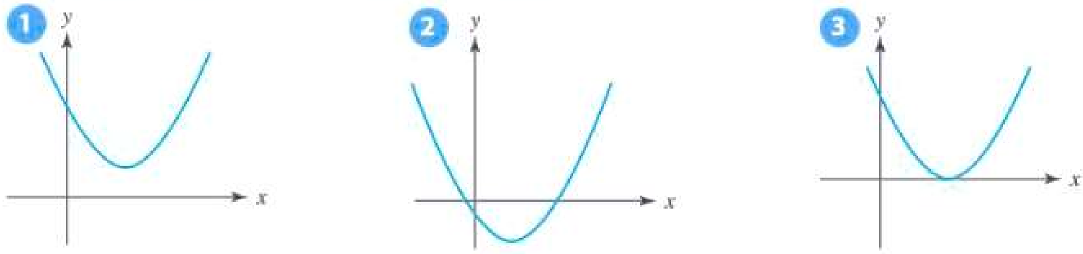
Use the quadratic formula to solve each of these equations.

a $7x^2+3x-8=0$ **b** $-x^2+4x-2=0$ **c** $x^2-12x+4=0$

Inside the square root of the quadratic formula you have the expression $b^2 - 4ac$. This expression is called the **discriminant**. You can use the discriminant to determine how many roots the equation has.

Key point

1. If $b^2 - 4ac < 0$ then the equation has no real roots.
2. If $b^2 - 4ac > 0$ then the equation has two real roots.
3. If $b^2 - 4ac = 0$ then the equation has one real root.



1. The curve does not cross the x-axis so the discriminant is negative.
2. The curve crosses the x-axis twice so the discriminant is positive.
3. The curve touches the x-axis once so the discriminant equals zero.

Example 2

Given that the quadratic equation $x^2 + 3x + k + 1 = 0$ has exactly one solution, find the value of k

$a = 1, b = 3, c = k + 1$

So $b^2 - 4ac = 3^2 - 4 \times 1 \times (k + 1)$ — Find the discriminant.

$= 5 - 4k$

$5 - 4k = 0 \Rightarrow k = \frac{5}{4}$ — The equation has exactly one solution so the discriminant is zero.

Task:

Work out how many real solutions each of these quadratic equations has.

a $x^2 - 5x + 7 = 0$ **b** $7 - 2x - 3x^2 = 0$ **c** $4x^2 - 28x + 49 = 0$

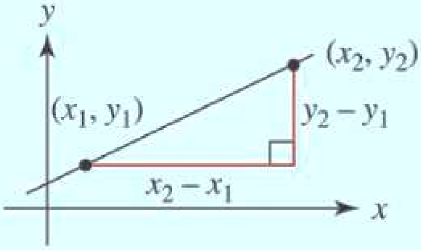
Topic G: Line graphs

This topic recaps how you can calculate key properties of straight line graphs when given two points on the line, in particular: the gradient, the length of a line segment, the midpoint of a line segment, the equation of the perpendicular bisector of a line segment, and the equation of the line.

The gradient of a line is a measure of how steep it is.

Key point

The gradient, m, of a line between two points (x_1, y_1) and (x_2, y_2) is given by $m = \dfrac{y_2 - y_1}{x_2 - x_1}$



Example 1

Calculate the gradient of the line through the points $A(1, -6)$ and $B(-5, 2)$

$$m = \frac{2-(-6)}{(-5)-1}$$

Use $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ with $x_1 = 1$, $x_2 = -5$ and $y_1 = -6$, $y_2 = 2$

$$= \frac{8}{-6}$$

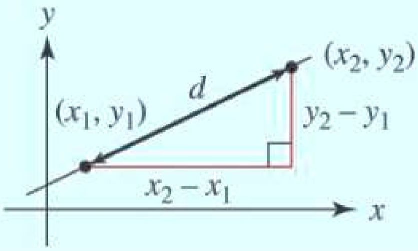
$$= -\frac{4}{3}$$

The line has a negative gradient so slopes down from left to right.

You also can find the length of a line segment, d, between two points using Pythagoras' theorem.

Key point

The length of the line segment, d, between two points (x_1, y_1) and (x_2, y_2) is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



Example 2

Calculate the exact distance between the point $(5, 1)$ and $(6, -4)$

$$d = \sqrt{(6-5)^2 + (-4-1)^2}$$

Use $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ with $x_1 = 5$, $x_2 = 6$ and $y_1 = 1$, $y_2 = -4$

$$= \sqrt{1^2 + (-5)^2}$$

$$= \sqrt{26}$$

Leave answer as a surd since this is exact.

Key point

The midpoint of the line segment from (x_1, y_1) to (x_2, y_2) is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Example 3

The points A and B have coordinates $(-4, -9)$ and $(6, -2)$ respectively. Find the midpoint of AB

$$\text{Midpoint} = \left(\frac{(-4)+6}{2}, \frac{(-9)+(-2)}{2}\right)$$
$$= \left(\frac{2}{2}, \frac{-11}{2}\right)$$
$$= (1, -5.5)$$

Use $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ with $x_1 = -4$, $x_2 = 6$ and $y_1 = -9$, $y_2 = -2$

Task:

Find the gradient of the line through each pair of points.

a (3, 7) and (2, 8) **b** (5, 2) and (–4, –6)

Calculate the exact distance between each pair of points.

a (8, 4) and (1, 3) **b** (– 3, 9) and (12, –7)

Find the coordinates of the midpoint of each pair of points.

a (3, 9) and (1, 7) **b** (2, –4) and (–3, –9)

You need to be able to work with equations of straight lines.

Key point

The equation of a straight line is $y = mx + c$ where m is the gradient and c is the y-intercept.

Example 4

Work out the gradient and the y-intercept of each of these lines.

a $y = \frac{1}{2}x + 4$ **b** $y + x = 5$ **c** $-2x + 3y + 7 = 0$

a Gradient $= \frac{1}{2}$ and y-intercept $= 4$

Since $y = mx + c$ where m is the gradient and c is the y-intercept.

b $y = 5 - x$

Rearrange to make y the subject.

So gradient $= -1$ and y-intercept $= 5$

c $3y = -7 + 2x$

Rearrange to make y the subject.

$y = -\frac{7}{3} + \frac{2}{3}x$

So gradient $= \frac{2}{3}$ and y-intercept $= -\frac{7}{3}$

Task:

Work out the gradient and the y-intercept of these lines.

a $y = 7x - 4$ **b** $y + 2x = 3$ **c** $x - y = 4$ **d** $3x + 2y = 7$

Example 5

Find the equation of the line through the points (3, 7) and (4, −2) in the form $y=mx+c$

$m=\frac{(-2)-7}{4-3}$

$=-9$

So the equation is $y-7=-9(x-3)$

$y-7=-9x+27$

$y=-9x+34$

First use $m=\frac{y_2-y_1}{x_2-x_1}$ to find the gradient.

Use $y-y_1=m(x-x_1)$ with $(x_1, y_1)=(3, 7)$, or you could use the point (4, −2) instead.

Expand the brackets and rearrange to the correct form.

Example 6

The line l_1 has equation $2x+6y=5$. The line l_2 is parallel to l_1 and passes through the point (1, −5). Find the equation of l_2 in the form $ax+by+c=0$ where a, b and c are integers.

$l_1: 2x+6y=5 \Rightarrow 6y=5-2x$

$\Rightarrow y=\frac{5}{6}-\frac{2}{6}x$

Rearrange to make y the subject so you can see what the gradient is.

The gradient of l_1 is $-\frac{2}{6}$ which simplifies to $-\frac{1}{3}$

Therefore the gradient of l_2 is $-\frac{1}{3}$

Since l_1 and l_2 are parallel.

So the equation of l_2 is $y-(-5)=-\frac{1}{3}(x-1)$

Use $y-y_1=m(x-x_1)$ to write the equation of l_2

$\Rightarrow y+5=-\frac{1}{3}(x-1)$

$\Rightarrow -3y-15=x-1$

Multiply both sides by −3 so that all coefficients are integers.

$\Rightarrow x+3y+14=0$

Rearrange to the correct form.

Example 8

The line l_1 has equation $7x+4y=8$. The line l_2 is perpendicular to l_1 and passes through the point (7, 3). Find the equation of l_2 in the form $ax+by+c=0$ where a, b and c are integers.

$l_1: 7x+4y=8 \Rightarrow 4y=-7x+8$

$\Rightarrow y=-\frac{7}{4}x+2$

Rearrange to make y the subject so you can see what the gradient is.

So the gradient of l_1 is $-\frac{7}{4}$ and the gradient of l_2 is $\frac{4}{7}$

Since $\left(-\frac{7}{4}\right)\times\frac{4}{7}=-1$

So the equation of l_2 is $y-3=\frac{4}{7}(x-7)$

Use $y-y_1=m(x-x_1)$ to write the equation of l_2

$\Rightarrow 7y-21=4(x-7)$

Multiply both sides by 7 so that all coefficients are integers.

$\Rightarrow 7y-21=4x-28$

$\Rightarrow 4x-7y-7=0$

Rearrange to the correct form.

Task:

Find the equation of the line through each pair of points.

a (2, 5) and (0, 6) **b** (1, –3) and (2, –5) **c** (4, 4) and (7, –7)

The line l_1 has equation $y = 5x + 1$

a Find the equation of the line l_2 which is parallel to l_1 and passes through the point (3, –3)

b Find the equation of the line l_2 which is perpendicular to l_1 and passes through the point (–4, 1)

The line l_1 has equation $y = 3 + \frac{1}{2}x$

a Find the equation of the line l_2 which is parallel to l_1 and passes through the point (–1, 5)

b Find the equation of the line l_2 which is perpendicular to l_1 and passes through the point (6, 2)

Topic H: Circles

Key point

A circle of radius r and centre (a, b) has equation $(x-a)^2+(y-b)^2=r^2$

Example 1

a Find the centre and radius of the circle with equation $(x-5)^2+(y+1)^2=9$

b Write the equation of a circle with centre $(-3, 7)$ and radius 4

a The centre is at $(5,-1)$

Equation is $(x-5)^2+(y-(-1))^2=9$ so $a=5$ and $b=-1$

The radius is $\sqrt{9}=3$

Remember to find the positive square root.

b $a=-3$, $b=7$ and $r=4$

So equation is $(x+3)^2+(y-7)^2=16$

Remember to square the radius.

Task:

Write the equations of these circles.

a circle with radius 7 and centre $(2, 5)$

b circle with radius 4 and centre $(-1, -3)$

c circle with radius $\sqrt{2}$ and centre $(-3, 0)$

d circle with radius $\sqrt{5}$ and centre $(4, -2)$

Find the centre and the radius of the circles with these equations.

a $(x-5)^2+(y-3)^2=16$

b $(x+3)^2+(y-4)^2=36$

c $(x-9)^2+(y+2)^2=100$

If you have the equation of a circle in expanded form then you can complete the square, as shown in Topic D, to write it in the form $(x-a)^2+(y-b)^2=r^2$ which will enable you to state the centre and radius.

Example 2

Find the centre and radius of the circle with equation $x^2+y^2-8x+4y+2=0$

$x^2-8x+y^2+4y+2=0$ — Group the terms involving x and the terms involving y

$(x-4)^2-16+(y+2)^2-4+2=0$ — Complete the square for x^2-8x and y^2+4y

$(x-4)^2+(y+2)^2=18$

So the centre is $(4,-2)$ and the radius is $\sqrt{18}=3\sqrt{2}$

Task:

Find the centre and the radius of the circles with these equations.

a $x^2+2x+y^2=24$

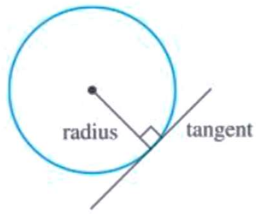
b $x^2+y^2+12y=13$

d $x^2+y^2+6x+8y+2=0$

e $x^2+y^2-8x-10y=3$

A **tangent** to a circle is a line which is perpendicular to a radius of the circle. Note that a tangent will intersect a circle exactly once.

You can use these facts to find the equation of a tangent to a circle.



Example 4

A circle has equation $(x+3)^2+(y-7)^2=26$

a Show that the point $(-4, 2)$ lies on the circle.

b Find the equation of the tangent to the circle that passes through the point $(-4, 2)$

a $(-4+3)^2+(2-7)^2=(-1)^2+(-5)^2$

$=1+25$

$=26$ so $(-4, 2)$ lies on the circle.

Substitute $x=-4$, $y=2$ into the equation.

b Centre of circle is $(-3, 7)$

Gradient of radius is $\dfrac{2-7}{-4-(-3)}=\dfrac{-5}{-1}=5$

Use $m=\dfrac{y_2-y_1}{x_2-x_1}$

A tangent is perpendicular to a radius so gradient of tangent is $-\dfrac{1}{5}$

Since $\left(-\dfrac{1}{5}\right)\times 5=-1$

Therefore equation of tangent is $y-2=-\dfrac{1}{5}(x+4)$

Use $y-y_1=m(x-x_1)$ with $(x_1, y_1)=(-4, 2)$

Task:

A circle has equation $(x-1)^2+(y+1)^2=10$. Find the equation of the tangent to the circle through the point $(2, -4)$. Write your answer in the form $ax+by+c=0$ where a, b and c are integers.

A circle has equation $x^2+(y-8)^2=153$. Find the equation of the tangent to the circle through the point $(3, -4)$. Write your answer in the form $y=mx+c$